# Tutorial 3 : Symbolic Reachability & ω-Regular Languages

CS60030 Formal Systems

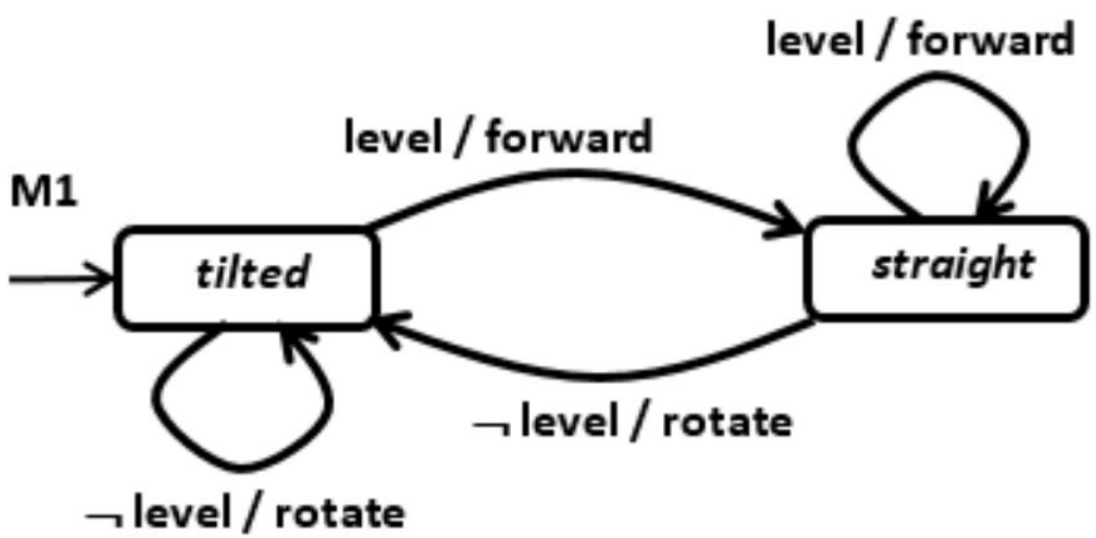
PALLAB DASGUPTA,
FNAE, FASc,
A K Singh Distinguished Professor in AI,
Dept of Computer Science & Engineering
Indian Institute of Technology Kharagpur
Email: pallab@cse.iitkgp.ac.in
Web: http://cse.iitkgp.ac.in/~pallab

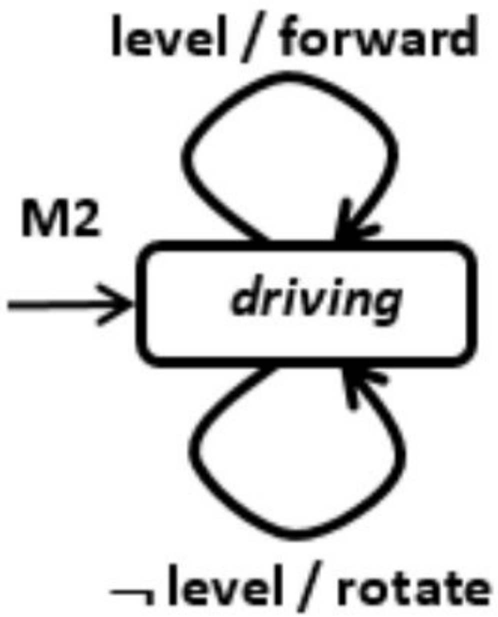

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

FORMAL METHODS FOR SAFETY CRITICAL SYSTEMS

# Characteristics Function

**Consider the finite state machines M1 and M2 for a robot. The variable, level, is an input to the machines, and the variables, forward and rotate, are outputs of the machines. Show the characteristic function representations of the transition relations of M1 and M2. You may use the first letter of each variable as a short form for convenience.**

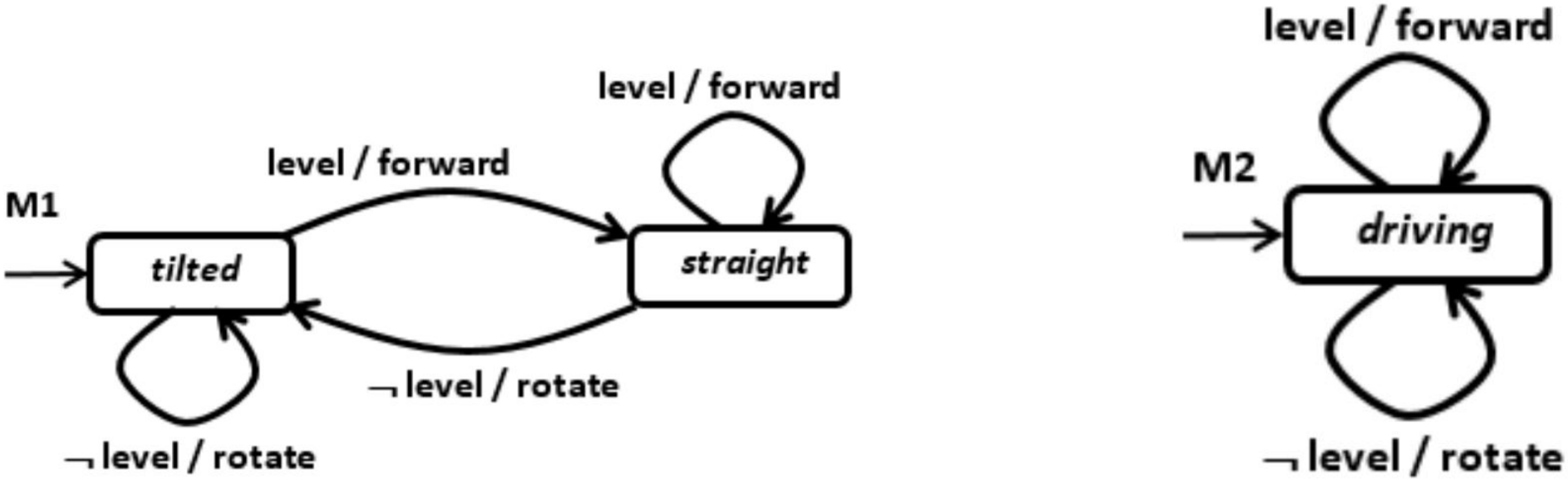


# Characteristics Function

1) Two FSMs are called simulation equivalent or language equivalent if they produce a similar output sequence when executed with the same input sequence. Develop a symbolic methodology for checking whether any two given FSMs are simulation equivalent.

2) Use the characteristic functions of previous slide to prove whether M1 and M2 are simulation equivalent. In this case, do we need to reach a fix point?



# Characteristic Function : Gray Counter

a) Consider a 3-bit counter whose counting sequence is shown below.

000 → 001 → 011 → 010 → 110 → 111 → 101 → 100 → 000 ….

Here the state is represented by a vector $\langle x_1, x_2, x_3 \rangle$ of 3 state variables. Let $\langle x_1', x_2', x_3' \rangle$ denote the next state.

1) Develop the characteristic function, $cf(x_1, x_2, x_3, x_1', x_2', x_3')$, representing the transition relation of the counter.

# Characteristic Function : Gray Counter

**The equations for $x_1'$, $x_2'$, and $x_3'$ can be written as**

- $x_1' = \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3 + x_1 \bar{x}_2 x_3 = x_2 \bar{x}_3 + x_1 x_3$
- $x_2' = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 = x_2 \bar{x}_3 + \bar{x}_1 x_3$
- $x_3' = \bar{x}_1 \bar{x}_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3 = x_1 x_2 + \bar{x}_1 \bar{x}_2$

**Therefore,** $cf = [x_1' \odot (x_2 \bar{x}_3 + x_1 x_3)][x_2' \odot (x_2 \bar{x}_3 + \bar{x}_1 x_3)][x_3' \odot (x_1 x_2 + \bar{x}_1 \bar{x}_2)]$

# Symbolic Reachability : Gray Counter

b) We wish to determine whether the counter is a Gray counter. For this purpose we need to check from the transition relation of part (a) that successive states differ in only one bit. Prepare a Boolean formula, , such that the satisfiability of  will enable you to determine whether the transition relation is one for the Gray counter?

# Symbolic Reachability : Gray Counter

$$\psi = [(x_1' = \overline{x_1}) \leftrightarrow (x_2' = x_2)(x_3' = x_3)] \wedge$$
$$[(x_2' = \overline{x_2}) \leftrightarrow (x_1' = x_1)(x_3' = x_3)] \wedge$$
$$[(x_3' = \overline{x_3}) \leftrightarrow (x_1' = x_1)(x_2' = x_2)]$$

$$\therefore \phi = \neg\psi \wedge cf$$

# NBA and ω-Regular Languages

Consider the language over the set of infinite words over Σ such that A occurs infinitely often and between any two successive A's there are an odd number of B's.

1) Represent this language in ω-Regular expression
2) Draw an NBA for this language

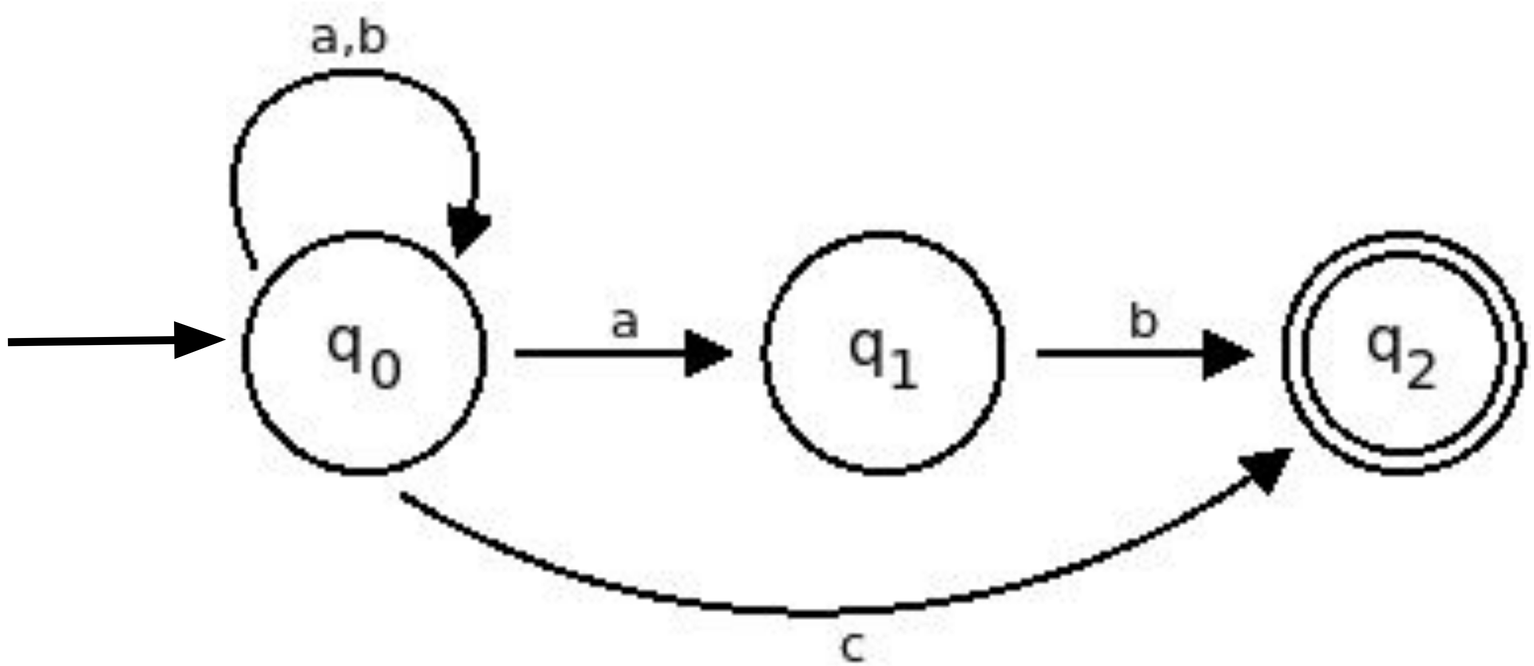
# NBA CONSTRUCTION

**Construct NBA for the following properties/expressions**

1. $(A^*C)^\omega$
2. $(AB + C)^* ((AA + B)C)^\omega$
3. Between two neighboring A's there are odd no. of B's
4. If A occurs, it occurs consecutively in multiples of three ($\Sigma$=(A,B))
5. $(A^*CA + BB)^*(A + CC)^\omega$

# NFA to NBA CONSTRUCTION

**Write the omega regular expression for the language accepted by the given NFA. Construct a NBA from the NFA accepting the omega regular language.**



# ω-Regular Languages

Write the ω-Regular Language for the following sentences:

1. A and B always alternate starting with A. This means only A is true in the first step, then only B is true in the next step, and this alternation between A and B is always repeated.

2. Between two neighboring A's there is at least one B.

3. Never is it that an A is followed by a B unless the A is immediately preceded by a C

4. If at some point C holds and at all points before it A did not hold and B held, then at some point after C, A and B both hold.

# Equivalence checking

**For each of the following pairs, determine if they are equivalent. If not, provide a counterexample**

1. $(A^*B)^\omega$ and $A^*.B^\omega$
2. $(A^* + B^*)^*.C^\omega \equiv A^*.C^\omega + B^*.C^\omega$
3. $(A^* + B.C)+.(C.C^*)^\omega$ and $(A^* + B.C)+.(C)^\omega$

# GNBA TO NBA

**Draw the NBA for the following GNBA, where F={{q1},{q2}}.**